Active fault-tolerant control of a Quadrotor UAV against actuator faults based on backstepping technique and adaptive observer

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**Abstract:** This paper addresses the stabilization problem of an underactuated quadrotor UAV system in presence of actuator faults. First, a dynamic model of the quadcopter was established using a Lagrange approach. High-order non-holonomic constraints as well as different physical phenomena that can influence the dynamics of the structure have been taken into account. Then, for actuator faults, a new active fault tolerance strategy based on a backtracking approach and an adaptive observer is developed. The simulation results obtained illustrate the ability of the proposed control strategy to maintain performance and preserve stability in the event of actuator failure.

### Introduction

In light of increasing needs regarding automated systems availability, safety, and performance, it is necessary to develop a diagnostic module to detect faults that may damage these systems operations and identify their origin or source.

Despite the tangible progress made, researchers must still deal with severe difficulties in controlling such systems, particularly in the presence of faults. Especially in the case of some systems like unmanned aerial vehicles (UAV).

Quadcopters have been the subject of several studies in particular in the field of diagnosis and fault tolerance. (Freddi, Longhi and Monteriù 2010), (Xulin and Yuying 2018), (Ouadine, et al. 2020), (Xiao-Lu 2020)

The work in (Avram, Zhang and Muse 2018) presents a nonlinear robust adaptive fault-tolerant altitude and attitude tracking scheme to accommodate actuator faults in a quadrotor. In (Xulin and Yuying 2018), the authors present a fuzzy active disturbance rejection control method for controlling a quadrotor UAV with actuator faults.

An active fault-tolerant tracking control system approach for actuator faults on a quadrotor was discussed in (Zhong, Liu and Zhang). A fault-tolerant controller was designed on basis of adaptive estimation for actuator faults in (Hasanshahi, Ahmadi and Amjadifard). In (Hong-Jun , Yanli and Tianbo ), the authors present the diagnosis and compensation of sensors and actuators faults in a quadrotor UAV based on a nonlinear high-gain observer. Other strategies are proposed in (Yujiang , Youmin and Wei), (Xiao-Lu), (Lien, Chao-Chung and Yi-Hsuan ).

In the field of active FTC, observer-based reconstruction and defect estimation (FRE) has gained increased interest in the last two decades. Its advantage is that it can estimate the defects without going through the residual generation phase. Various observer-based FRE design techniques have been presented in the literature, mainly based on sliding-mode observers, observers for singular systems, and adaptive observers (Jiang and Yu 2012). When faults are modeled in terms of parameter changes, adaptive observers can be used to estimate these faults.

This article presents a new fault-tolerant active control technique on a quadrotor in the presence of actuator faults. It is based on a joint use of an adaptive observer for fault reconstruction and estimation and a backstepping approach for system control. Compared to previous work on the active FTC of a quadcopter UAV, in our work we have not neglected the non-linearity of the dynamic model of the quadcopter and the high-order non-holonomic constraints. It was used an adaptive observer proposed in (Oucief, Tadjine, & Labiod, 2016a)

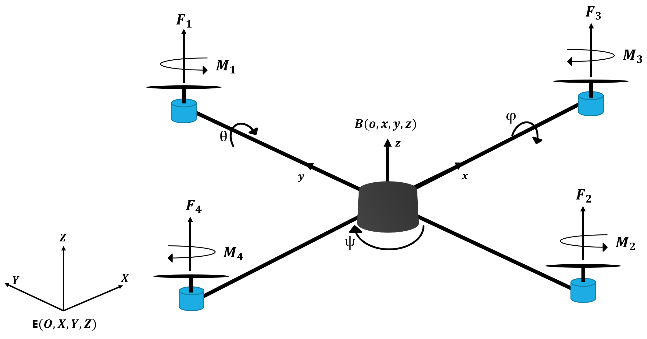
In the first section, the dynamic modeling of the quadcopter is carried out. Then, in the second section, a robust control strategy with actuator faults is established based on the backstepping technique. To detect defects, an adaptive observer was developed to estimate the size of defects. Finally, in the last section, simulations on MATLAB were carried out to validate the synthesized control laws. The results were conclusive in the presence of faults in the actuators.

### Quadrotor Modelling

The aerial robot under study consists of a rigid cross frame coupled with four propellers, as illustrated in figure 1. The forward/ backward left/ right and the yaw movements are generated by a differential control strategy of the thrust delivered by each rotor. The up-down motion increases or reduces the overall thrust while keeping an equal individual thrust. To minimize the yaw drift induced by the responsive torques, the quadrotor aircraft is designed so that the set of rotors (right-left) spins clockwise and the set of rotors (front-rear) spins counter-clockwise.

Let E (O, X, Y, Z) designate an inertial frame, and B (o,x,y,z) designate a frame permanently coupled to the quadrotor, as illustrated in figure 1.

The absolute location is denoted by the three coordinates (x, y, z) and its attitude by the three Euler’s angles (ϕ, θ, ψ) respectively called Roll angle (*ϕ* rotation around x-axis), Pitch angle (*θ* rotation around y-axis) and Yaw angle (*ψ* rotation around z-axis).



**Figure 1.** Quadrotor configuration

Literally, by using formalism of Newton-Euler, the quadrotor complete model (position and orientation dynamic) is provided as in (Bouadi, Bouchoucha and Tadjine) by:

|  |  |  |
| --- | --- | --- |
|  |  | (1a) |
|  |  | (2b) |
|  |  | (3c) |
|  |  | (4d) |
|  |  | (5e) |
|  |  | (6f) |

Where:

* is the lift coefficient.
* drag coefficient.
* , and constant inertia
* , and are the translation drag coefficients.
* , and are the aerodynamic friction coefficients around (X,Y,Z).
* is the distance between the quadrotor center of mass and the rotation axis of propeller
* is the rotor inertia.

The quadrotor object of our study is (Draganfly IV: Manufactured by Draganfly Innovations). Parameter identification is studied in (Derafa, Madani and Benallegue 2006) (Table)

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

, , and are the control inputs of the system which are written according to the angular velocities of the four rotors as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (3) |
|  |  | (4) |

From the equations of the translation dynamics (1) we can deduce the expressions of the high-order nonholonomic constraints:

|  |  |
| --- | --- |
|  | (5a) |
|  | (5b) |

### Nonlinear adaptive observer design

#### State-space model

The complete model resulting by adding the actuator faults in the model (1a) can be written in the state-space form:

|  |  |  |
| --- | --- | --- |
|  |  | (13) |
|  |  |

with is the state vector of the system, such as:

|  |  |  |
| --- | --- | --- |
|  |  | (14) |

and , is the resultant vector of actuator faults related to quadrotor motions, with represent the actuators faults vector.

is the input control vector,

is the output vector, represent the actuators faults vector.

and  are known nonlinear function vectors and the regressor is a known function matrix which may depend nonlinearly on . , , and are known constant matrices. Finally, the output vector is giving by .

Throughout this paper, the system model (13) has to satisfy the following conditions:

**C0:** The pair (C, A) must be observable;

**C1:** The vector function is continuous in its variables;

**C2:** and satisfy the Lipschitz property with respect to , i.e., there exist positive constants and such that:

|  |  |  |
| --- | --- | --- |
|  |  | (15a) |
|  |  | (15b) |

**C3:** The fault vector is piecewise constant and bounded in the following sense:

Were

|  |  |  |
| --- | --- | --- |
|  |  | (16) |

is a known constant vector and is a known positive constant.

#### The adaptive state observer

The typical form of the adaptive state observer dealing with the class of nonlinear systems (13) is given by the following equations ( (Cho and Raramani), (That and Ding)):

|  |  |  |
| --- | --- | --- |
|  |  | (17a) |
|  |  | (17b) |

where is the state estimate, . is the unknown parameter vector estimate.

is the observer gain, is a positive constant and is a matrix to be designed.

Under conditions 1, 2 and 3, the state estimate converges to the actual state and converges to , if there exist a symmetric positive definite matrix and a matrix such that (Cho and Raramani):

|  |  |  |
| --- | --- | --- |
|  |  | (18) |

Equality (18) is known as the observer matching condition (Floquet, Edwards and Spurgeon), it hold if and only if ( (Corless and Tu); (Raoufi , Jose Marquez and Solo)):

|  |  |  |
| --- | --- | --- |
|  |  | (19) |

It is easy to check that the observer matching condition (18) is not satisfied for our system and hence no adaptive observer of the form presented in (Cho and Raramani) and (That and Ding) can be used to estimate and .

In (Oucief, Tadjine and Labiod , A new methodology for an adaptive state observer design for a class of nonlinear systems with unknown parameters in unmeasured state dynamics), authors have developed an adaptive observer for a certain class of non-linear systems. This observer employs the nonlinear system model described by equation (13).

For developing the considered adaptive observer, in addition to conditions C0, C1, C2 and C3 the system model (13) has to satisfy the following conditions:

**C4:** The matrices A, B, E and C satisfy the following conditions:

|  |  |  |
| --- | --- | --- |
|  |  | (20a) |
|  |  | (20b) |
|  |  | (20c) |

**C5:** The first derivative in time of is continuous and bounded provided that is bounded.

To satisfies the condition C4 giving by equation (20), the state space is rearranged. From equations (1) and considering the actuator faults, we obtain:

|  |  |  |
| --- | --- | --- |
|  |  | (21a) |
|  |  | (21b) |
|  |  | (21c) |
|  |  | (21d) |
|  |  | (21e) |
|  |  | (21f) |
|  |  | (21g) |
|  |  | (21h) |
|  |  | (21i) |
|  |  | (21j) |
|  |  | (21k) |
| ² |  | (21l) |

With

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | (22) |
|  |  |  |  |  |
|  |  |  |  |  |

When these conditions are satisfied, a stable observer for the system (13) has the form (Oucief, Tadjine and Labiod , A new methodology for an adaptive state observer design for a class of nonlinear systems with unknown parameters in unmeasured state dynamics):

|  |  |  |
| --- | --- | --- |
|  |  | (24a) |
|  |  | (24b) |
|  |  | (24c) |

( is the unknown parameter vector).

H and G are constant matrices to be designed and is the learning rate matrix.

A sufficient condition for the asymptotic stability of the adaptive state observer is described in the following theorem.

**Theorem 2.** (Oucief, Tadjine and Labiod , A new methodology for an adaptive state observer design for a class of nonlinear systems with unknown parameters in unmeasured state dynamics)

Under conditions 0, 1, 2, 3, 4 and 5, the estimate of the state converges to the real state asymptotically while converges to if there are positive real constants ε1 and ε2 and matrices , H and G such that :

|  |  |  |
| --- | --- | --- |
|  |  | (25a) |
|  |  | (25b) |

However, condition C4 is satisfied, and as a consequence, the adaptation law (24) is feasible. Notice that according to the structure of the matrices C and E, the system is already in the form (13) and therefore no state transformation is needed to design the adaptive observer.

As the state, input, fault and output vector, respectively, then the system described in (21) can be rewritten as a state space system of the form (13), where

|  |  |  |
| --- | --- | --- |
|  |  | (23a) |
|  |  | (23b) |
|  |  | (23c) |

#### Observer design

The objective of this part is to synthesize an adaptive state observer corresponding to the model quadrotor given by (23) which can be described in the form of system given by (13).

Since is a matrix, the computation of its Lipschitz constant is not a trivial task. In order to do this, we evaluate the Lipschitz constants of the individual rows of .

|  |  |
| --- | --- |
|  | (26c) |

The computation of is given by

|  |  |  |
| --- | --- | --- |
|  |  | (27) |

We found that we can take .

The function is given by:

|  |  |
| --- | --- |
|  | (28) |

However, is locally Lipschitz, which means that its Lipschitz constant depends on the region where the system operates. Suppose that the control input is chosen such that it keeps the state bounded in the set

|  |  |
| --- | --- |
|  | (29) |

Then can be extended into the bounded functionwhich coincides with in , and thus the system becomes globally Lipschitz (Farza, M'Saad and Maatoug), can be computed using the relation . The computation of is given by

|  |  |
| --- | --- |
|  | (30) |

Where . Which yields .

Since is a matrix, the computation of is given by

|  |  |  |
| --- | --- | --- |
|  |  | (31) |

Where are the known constant vector. Finally, the bound in conditions C3 can be chosen as .

We start the observer design by computing the observer gains presented in (24). Let and . Finding the observer gains by solving (25a) under equality constraint (25b) is not a trivial task. To make this problem easily tractable, we intend in this subsection to transform it into an LMI optimization problem. By introducing the following change of variable

|  |  |
| --- | --- |
|  | (32) |

where and using the Schur complement (Boyd, El Ghaoui and Feron), inequality (25a) can be turned into the following LMI in the variable P

|  |  |
| --- | --- |
|  | (33) |

where .

Also, by using the same idea used in (Corless and Tu) we can turn the problem of the resolution of equality (15b) into the following LMI optimization problem.

Minimize subject to

|  |  |
| --- | --- |
|  | (34) |

Where , and is a positive scalar. Therefore, computing ,, and involves solving LMIs (21) and (22), simultaneously, for , .

The LMIs were solved using CVX, a Matlab-based modelling system for convex optimization (Grant , Boyd and Ye). we obtain the following results

|  |  |
| --- | --- |
|  | (35) |

Where

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  | , |  |
|  |  |  |
|  |  | (36) |
|  |  | (37) |
|  |  |  |
|  |  |  |

The unknown parameter vector estimates and confirming to (24) is then

|  |  |  |
| --- | --- | --- |
|  |  | (38) |
|  | | (39) |

Where and are the tracking-errors of andrespectively. and .

### Control Strategy of Quadrotor with Actuator Faults

#### Control Strategy

Generally, a fault-tolerant system is composed of two cascaded modules. The first one is a monitoring module which is used to detect faults, and diagnose their location and significance in a system. The second is a recovery module taking necessary actions so that the faulty system can achieve the control objectives almost at any time (Jain, J. Yamé and Sauter).

In our case, an adaptive observer is used like a monitoring module (Figure 2) and the recovery module is based on the Backstepping approach.

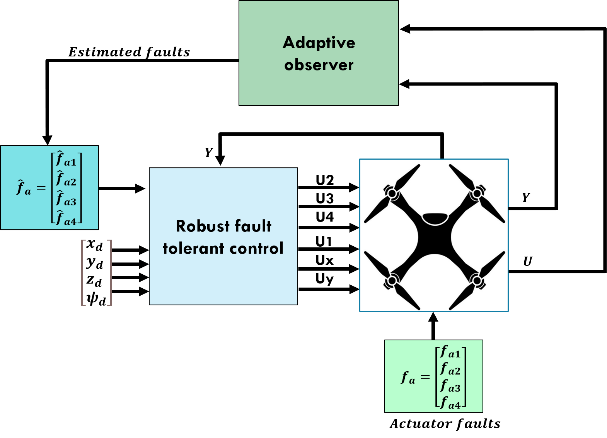


Figure 2. Fault-tolerant control system architecture

The following assumptions are needed for the analysis:

**Assumption 1: (justifier physiquement toutes ces hypothèses)**

The resultant of actuator faults related to quadrotor motions are slowly varying in time and bounded, as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (40) |

Where {,,,} are positive constants.

**Assumption 2:**

The unknown’s parts including the resultants of actuator faults related to the quadrotor motions are also bounded:

|  |  |
| --- | --- |
|  | (41) |

Where {,,,} are positive constants.

The proposed control approach is based on two loops (internal loop and external loop). The internal loop has four control laws: control of roll, control of pitch, control of yaw, and control of altitude. The external loop has two control laws of coordinates and .

The external control loop produces the desired roll () and pitch () via the corrective block (shown by equation (5)). The corrective block has as a goal to correct the rotation of the roll and pitch based on the desired yaw (). The synoptic scheme (Figure 3) below illustrates this control strategy:

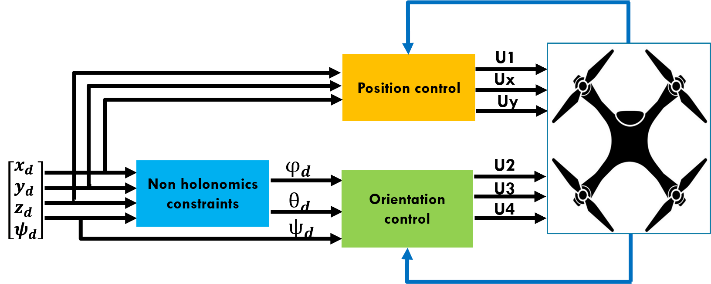


Figure 3. Synoptic scheme of the control strategy

#### Control laws

Based on the backstepping technique, an iterative algorithm is used to synthesize the control laws forcing the system to follow the desired path in presence of actuator failures, we summarize all stages of calculation concerning the tracking errors and Lyapunov functions in the following way:

|  |  |
| --- | --- |
|  | (42) |
|  |  |

The related Lyapunov functions are provided by:

|  |  |  |
| --- | --- | --- |
|  |  | (43) |

The synthesized stabilizing control laws are as described in the following:

|  |  |
| --- | --- |
|  | (44a) |
|  | (44b) |
|  | (44c) |
|  | (44d) |
|  | (44e) |
|  | (44f) |

**Proof**

Considering the first subsystem:

|  |  |
| --- | --- |
|  | (45) |

The corresponding reduced order observer is:

|  |  |
| --- | --- |
|  | (46) |

The calculation of the command is done in two steps.

***Step 1:*** For the first step we consider the first tracking-error given by

|  |  |
| --- | --- |
|  | (46) |

Let the first Lyapunov function candidate:

|  |  |
| --- | --- |
|  | (47) |

The time derivative of (47) is given by:

|  |  |
| --- | --- |
|  | (48) |

The stabilization of can be obtained by introducing a new virtual control :

|  |  |
| --- | --- |
|  | (49) |

The equation (48) is then

|  |  |
| --- | --- |
|  | (50) |

***Step 2:*** For the second step we consider the following tracking-error given by

|  |  |
| --- | --- |
|  | (51) |

Let and and from (40) notice that .

Then, from (13) and (24), it follows that

|  |  |
| --- | --- |
|  | (52) |
|  | (53) |

Where and .

The augmented Lyapunov function is given by:

|  |  |
| --- | --- |
|  | (54) |

The time derivative of is given by:

|  |  |
| --- | --- |
|  | (55) |

The time derivative of is given by:

|  |  |
| --- | --- |
|  | (56) |

Substituting (52) and (53) into (56) yields

|  |  |
| --- | --- |
|  | (57) |

Using (25b), we get:

|  |  |
| --- | --- |
|  | (58) |

By using the Lipschitz conditions (15a) to (15b), and inequality (16), we obtain the following inequalities

|  |  |
| --- | --- |
|  | (59) |

and

|  |  |
| --- | --- |
|  | (60) |

where and are positive constants. Substituting (59) and (60) into (58), we obtain

|  |  |
| --- | --- |
|  | (61) |

Substituting (61) into (55) yields

|  |  |
| --- | --- |
|  | (62) |

Where .

By replacing by the estimate the stabilization of (, ) can be obtained by introducing the input control :

|  |  |
| --- | --- |
|  | (63) |

The same steps are followed to extract , , , and .

### Simulation Results

To evaluate the performance of the controller proposed in this work, we executed test simulations in MATLAB. In the first test (Test 1), the motion of the quadrotor is considered normal without faults. In the second test (Test 2), we created multiple actuator failures relating to roll, pitch, yaw, and altitude motions.

Results without faults (Test 1) are shown in Figure 4, Figure 7, Figure 9 and Figure 11. The state estimates are shown in Figure 4.



Figure 4. Tracking simulation results of trajectories along roll (ϕ), pitch (θ), yaw angle (ψ), and Attitude Z axis (Test 1)

From these simulation results (Figure 4), it can be seen that the true and estimated state by using this adaptive observer are matched perfectly. As displayed in Figure 4, the estimation errors respectively of ϕ, θ, ψ and z are , , and , which clearly illustrates good performances and robustness towards stability and tracking of this control strategy with respect to the backstepping approach in the absence of faults.

In these tests (Test 2), four actuator faults related to roll, pitch, yaw and altitude motions (ϕ, θ, ψ, z) is simulated with 150% of maximum values of inputs control *, ,*  and 100% maximum value of input control between the instants 10s and 20s and start decreasing between 20s and 40s.

The results of these tests are shown in Figure 5, Figure 8, Figure 10 and Figure 12. The fault’s evolution and their estimates are given in Figure 5.



Figure 5. Fault estimation (Test 2)

According to Figure 5, there is very excellent estimation of the actuator faults, even the presence of transient peaks in the discontinuous points (10s, 20s, 30s and 40s). After fault injection in the motors, the estimate of and converges rapidly to the real values. Meanwhile, the estimate of converge rapidly after 0,4 s. As displayed in Figure 5, the estimation errors respectively of and are , , and .Therefore, the proposed observer can give a fast and accurate fault estimation.



Figure 6. Tracking simulation results of trajectories along roll (ϕ), pitch (θ), yaw angle (ψ), and Attitude Z axis (Test 2)

As displayed in Figure 6, there is very excellent tracking of the intended trajectories even after the appearance of actuator faults. Moreover, we can observe well the tracking of the planned trajectories, with small transient variations in roll, pitch, yaw, and altitude movements in the instants of occurrence of faults (10s, 20s, 30s and 40s).

Therefore, the estimation errors of roll (ϕ), pitch (θ) and yaw (ψ) still keep close to zero (), meanwhile, the estimation error of attitude Z still remain below after 0,2 m. Despite that, the trajectory tracking of our system is assured.

This figure illustrates the advantage of this method over FDI techniques is that the used observer for FRE can also be used for state feedback control because it is designed to preserve accurate state estimation even in the faulty case.



Figure 7. Error tracking results of trajectories along roll (ϕ), pitch (θ), yaw angle (ψ), and Attitude Z axis (Test 1)

Figure 7 illustrate the error tracking results of trajectories along roll (ϕ), pitch (θ), yaw (ψ), and attitude Z in absence of actuator faults (Test 1). We can see that the errors converge rapidly to 0 and remain below.

While, as displayed in Figure 8, the estimation errors of roll (ϕ), pitch (θ) and yaw (ψ) still keep close to zero (), meanwhile, the estimation error of attitude Z still remain below after 0,2 m which represent 4% of the desired attitude.

Figure 9 and Figure 10 illustrate the inputs control , *,*  of our system. It is easy to notice the transient peaks in all controllers.

Despite it, the stability of the closed-loop dynamics of the quadrotor is assured. Furthermore, we can observe input control signals provided by this control strategy (Test 1 and Test 2) are acceptable and physically realizable.



Figure 8. Error tracking results of trajectories along roll (ϕ), pitch (θ), yaw angle (ψ), and Attitude Z axis (Test 2)

|  |  |
| --- | --- |
| Figure 9. Control inputs of actuators in normal case (Test 1) | Figure 10. Control inputs of actuators in faulty case (Test 2) |

To judge the results obtained in the two tests numerically, we will compute two numerical criteria: the RMS (Root Mean Square) error and the error mean. The results obtained are presented in the table below (Table 1).

Table 1. Numerical evaluation of the control strategy in Test 1 and Test 2

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **RMS** | | | | **Error mean** | | | |
| **ϕ** | **θ** | **ψ** | **Z** | **ϕ** | **θ** | **ψ** | **Z** |
| **Test 1** |  |  |  |  |  |  |  |  |
| **Test 2** |  |  |  |  |  |  |  |  |

We note that the RMS for the orientation coordinates (ϕ, θ, ψ) is of the order of and of the order of for the Z altitude coordinate in faulty case (Test 2) and the same remark for the error mean in faulty case. The quantitative analysis confirms that the proposed strategy not only ensures a satisfactory tracking performance of the state estimation but also preserves a low energy consumption with small control inputs.

Figure 11 and Figure 12 illustrate the 3D trajectory of the quadrotor aircraft throughout the flight. The simulation results shown by this figure indicate high performances and resilience towards stability and tracking even after the occurrence of actuator faults (Figure 12), which shows the efficacy of the control method suggested in this work.

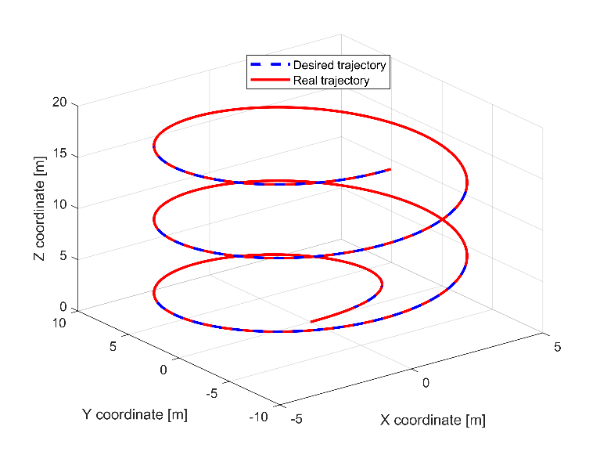


Figure 11. Global trajectory of the quadrotor in 3D along the (X, Y, Z) axis (Test 1)

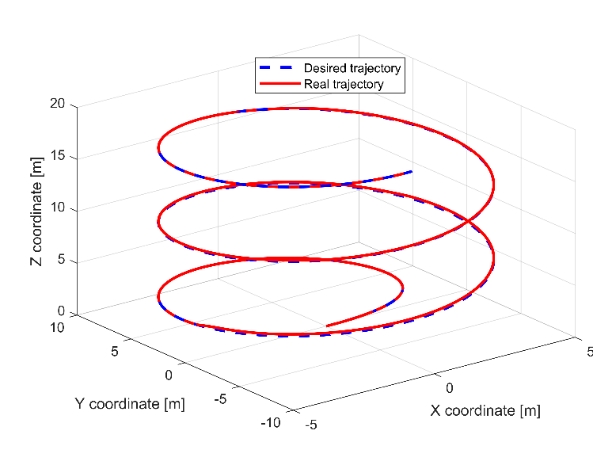


Figure 12. Global trajectory of the quadrotor in 3D along the (X, Y, Z) axis (Test 2)

### Conclusion

This paper presents a novel active fault-tolerant control strategy for diagnosing the actuator faults for a quadrotor vehicle. This approach is based on the observer-based fault reconstruction and estimation (FRE) technique using an adaptive observer. Firstly, we introduced a complete nonlinear dynamical model of the quadrotor, taking into consideration several physics phenomena that might impact our system's navigation in space. Secondly, we presented a stabilizing control law, in the presence of actuator faults, based on backstepping technique. Thirdly, an adaptive observer has been developed to estimate simultaneously the system state used in feedback control and actuator faults used in the FDI task.

Many test simulations in MATLAB have been executed to evaluate the performance of the proposed strategy. In the first test, the motion of the quadrotor is considered normal without faults. In the second test, we created multiple actuator failures relating to roll, pitch, yaw, and altitude motions. Simulation results clearly illustrates good performances and robustness towards stability and tracking of this control strategy with respect to the backstepping approach in the absence of faults. There is very excellent estimation of the actuator faults and the intended trajectories even after the appearance of actuator faults. We can see that the errors converge rapidly to 0 and remain below. While the estimation errors of roll (ϕ), pitch (θ) and yaw (ψ) still keep close to zero (), meanwhile, the estimation error of attitude Z still remain below 0,2 *m* which represent 4% of the desired attitude. Furthermore, we can observe input control signals provided by this control strategy are acceptable and physically realizable.

The contribution of this work, firstly, is the use of a complete model of the quadrotor considering the non-linearities and the high-order nonholonomic constraints of the system which gives a real behavior of the quadrotor, especially in faulty cases. Secondly, It’s the first use of the adaptive observer proposed in (Oucief, Tadjine, & Labiod , 2016a), in the field of active FTC for quadrotor UAV. This observer can estimate the system state and actuator faults simultaneously, which can be used respectively in feedback control and the FDI task of the actuator faults. Another advantage of the use of this observer structure lies in the fact that neither the conventional adaptive state observer nor any other alternative to the adaptive observer can be used in FTC in the case of our complete nonlinear dynamical model of the quadrotor UAV because of the non-satisfaction of the persistent excitation condition by model used. While using an adaptive observer as proposed in (Oucief, Tadjine, & Labiod , 2016a) does not require the system structure to satisfy the standard observer matching condition required in the conventional adaptive state observer. Finally, the use of these observers allows the estimation of any number of faults, regardless of the number of measured outputs, and it can estimate additive and multiplicative faults. The observer gains can be solved together with the Lyapunov inequality using LMI-based computations and do not require to change the system model into a special form like in (Oucief, Tadjine, & Labiod, 2016b) or the resolution of a system of partial differential equations like in (Stamnes, Aamo, & Kaasa, 2011).

This strategy can be easily applied to other nonlinear systems faults tolerant where several faults occur simultaneously. or the process itself in the case where sensor faults occur.

The simulation results have demonstrated the excellent effectiveness of this control strategy, and it maintains the stability and performance of the quadrotor even in the occurrence of actuator faults.

The observer can estimate the system state and actuator faults simultaneously. Neither the conventional adaptive state observer nor any other alternative to the adaptive observer can be used in FTC in the case of our complete nonlinear dynamical model of the quadrotor UAV because of the non-satisfaction of the persistent excitation condition (for more details see section 2.2).(reformule et expliquer d’avantage pourquoi ce n’est pas possible avec les autres méthodes?)

While using an adaptive observer as proposed in (Oucief, Tadjine, & Labiod , 2016a) does not require the system structure to satisfy the standard observer matching condition required in the conventional adaptive state observer. Thirdly, the use of these observers allows the estimation of any number of faults, regardless of the number of measured outputs, and it can estimate additive and multiplicative faults. The observer gains can be solved together with the Lyapunov inequality using LMI-based computations and do not require to change the system model into a special form or the resolution of a system of partial differential equations like in (Stamnes, Aamo, & Kaasa, 2011).(Reformuler c’est incompréhensible)

In this paper, an active fault-tolerant control strategy is considered. Trajectory tracking of a quadrotor in the presence of actuator faults is studied.

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